

FORCED RESPONSE OF MISTUNED BLADED DISKS

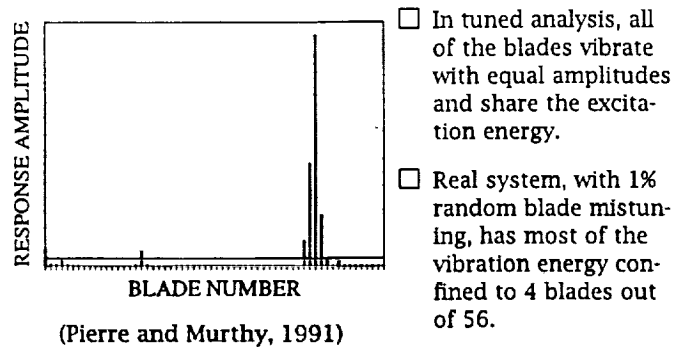
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Mistuning

- Manufacturing tolerances, material non-uniformities, non-identical root fixtures, and in-service degradation result in blade-to-blade differences that destroy cyclic symmetry
- *Small* mistuning can cause large, *catastrophic* changes in blade vibrational response
 - ☐ amplitudes of vibration of some blades may increase by several hundred percent, producing “rogue” blades and HCF failure
 - ☐ free and forced responses may be highly *sensitive* to mistuning
 - ☐ tuned system predictions may be *qualitatively* in error and grossly underestimate blade forced response and overestimate fatigue life
- A credible forced response prediction system for turbomachinery vibration must take mistuning into account

An Example of Mistuning Effects on the Free Aeroelastic Response

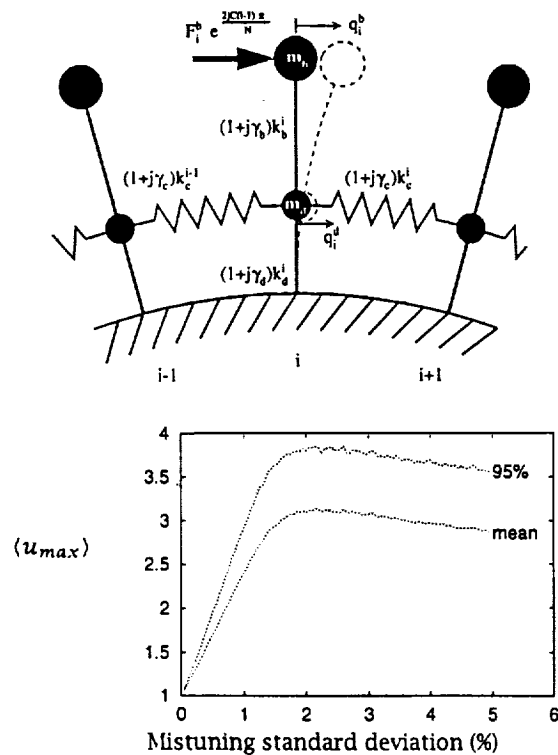


Mistuning causes *vibration localization*

- much larger amplitudes for some blades
- high stresses
- blade fatigue

If unaccounted for, mistuning could cause cracks and catastrophic blade failures.

Effect of mistuning on forced response for a common blade assembly model



Obstacles

- Mistuned assembly analyses are very expensive. Parametric studies cannot be performed

→ need for accurate reduced-order models

- Mistuning is random by nature

- ☐ mistuning pattern (and sometimes mistuning strength) is typically not available

- ☐ mistuning differs from rotor to rotor

- ☐ mistuning that results from in-service degradation cannot be modeled deterministically

→ calls for *statistical* and parametric tools

- Studies of mistuning by Afolabi, Bendiksen, Ewins, Griffin, Kaza, Kielb, Pierre, Sinha, Srinivasan, Mignolet, *etc.*, have led to general conclusions:

- ☐ helps flutter

- ☐ increases forced response amplitudes

- However — quantitatively and even qualitatively different findings regarding other issues

- ☐ blade with largest amplitude

- ☐ forced response amplitude increase over tuned system

- A new perspective of the mistuning problem (Bendiksen, Pierre):

- ☐ Mistuning belongs to the broader topic of repetitive structures with periodicity-breaking irregularities

- ☐ identification of the basic physical mechanism governing mistuning effects: sensitivity of aeroelastic eigen-solution to mistuning is inverse proportional to the distance between the eigenvalues

$$\delta^2 \lambda_j \propto \frac{1}{\lambda_{oj} - \lambda_{ok}}$$

- ☐ closeness of eigenvalues is governed by the *interblade coupling* and number of blades
 - weakly coupled assemblies are highly sensitive to mistuning (interblade coupling depends on frequency)
 - assemblies with many blades are more sensitive
 - mistuning effects increase with frequency (tip modes)
 - ☐ highly sensitive mistuned assemblies feature localized responses
- Formulation of a preliminary unifying theory of mistuning
 - Demonstration of the importance of considering mistuning effects at the design stage

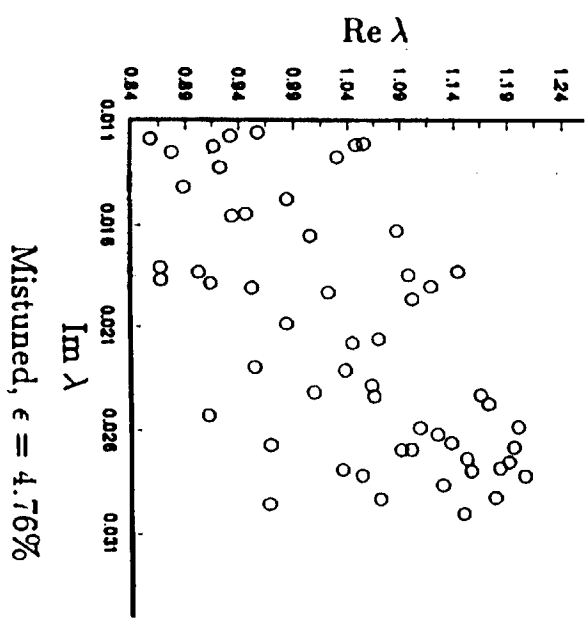
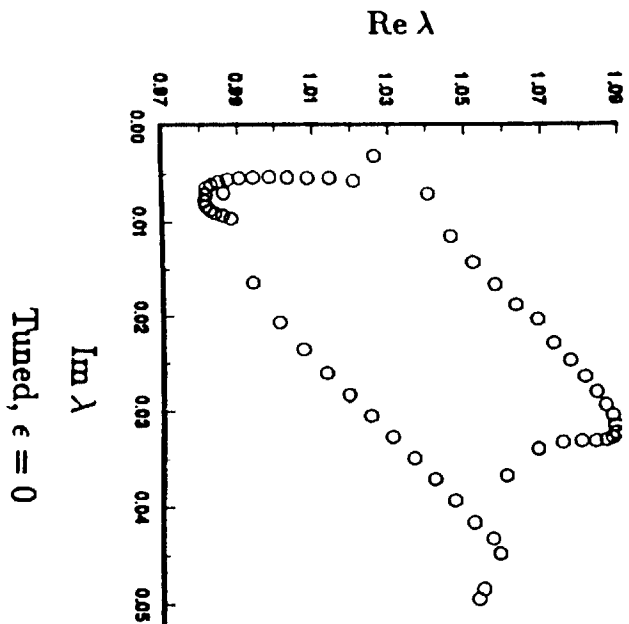
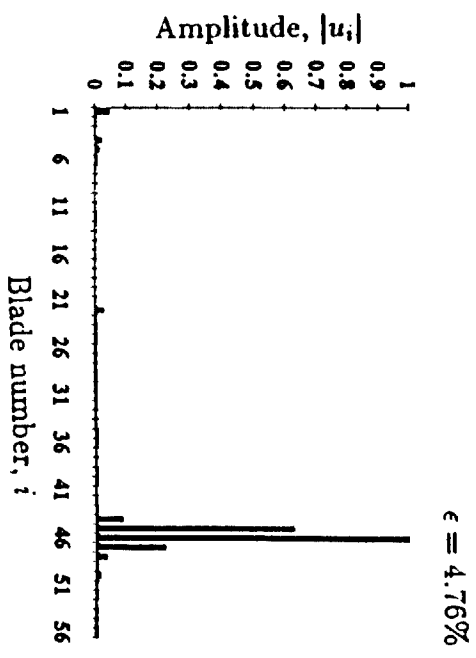
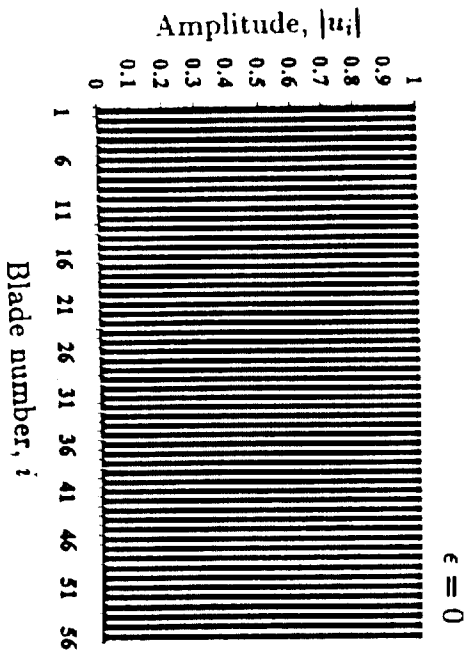
Objectives

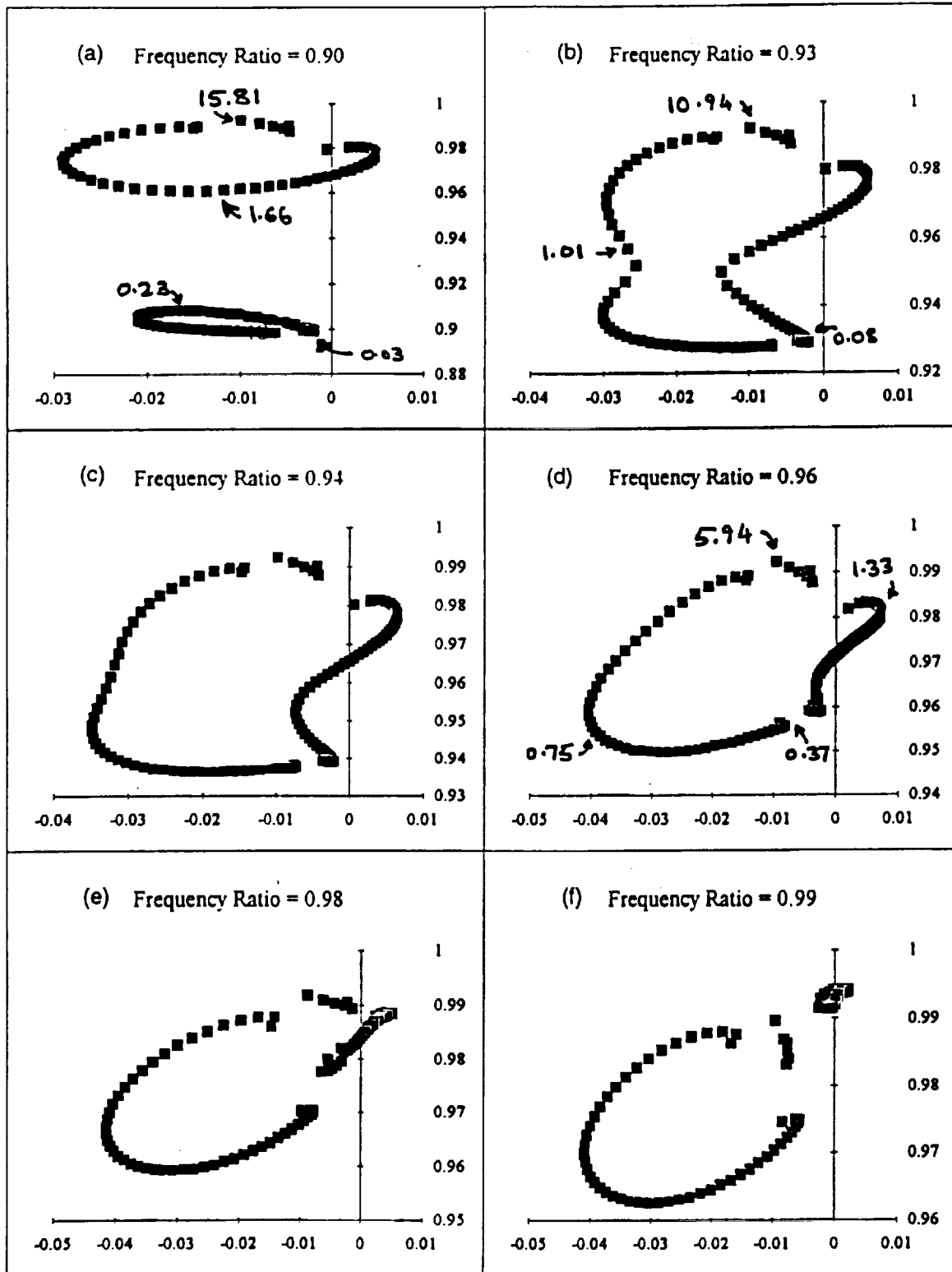
Provide the designer with tools for predicting the forced response amplitudes of real (*i.e.*, mistuned) bladed disks. Incorporate a mistuning analysis capability into forced response prediction system (FREPS)

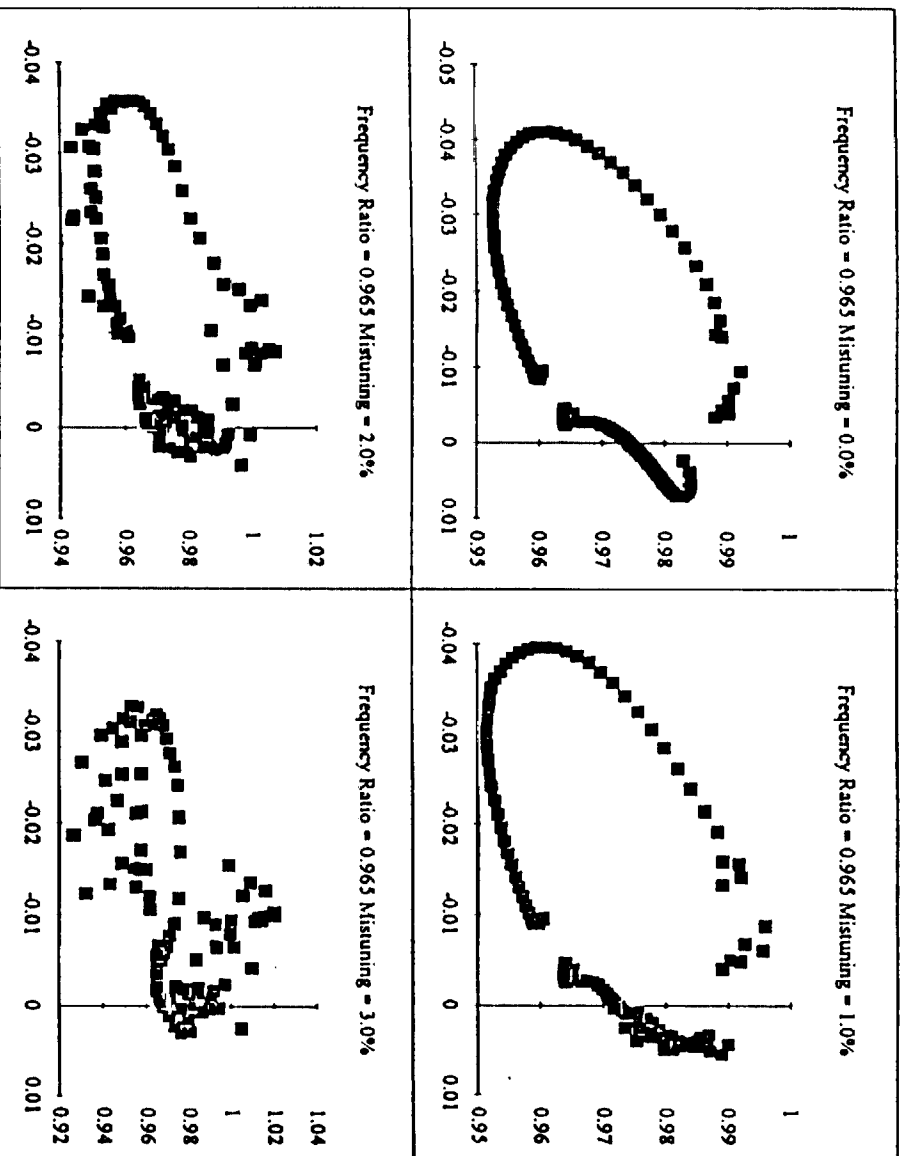
- develop low-dimensional reduced-order models
- evaluate the significance of mistuning effects in terms of system parameters. Identify key parameters governing sensitivity to mistuning.
- predict the sensitivity of the system dynamics to blade mistuning
- determine true response amplitudes for typical mistuned bladed disks
- obtain confidence intervals for amplitudes and stresses and estimates of fatigue life

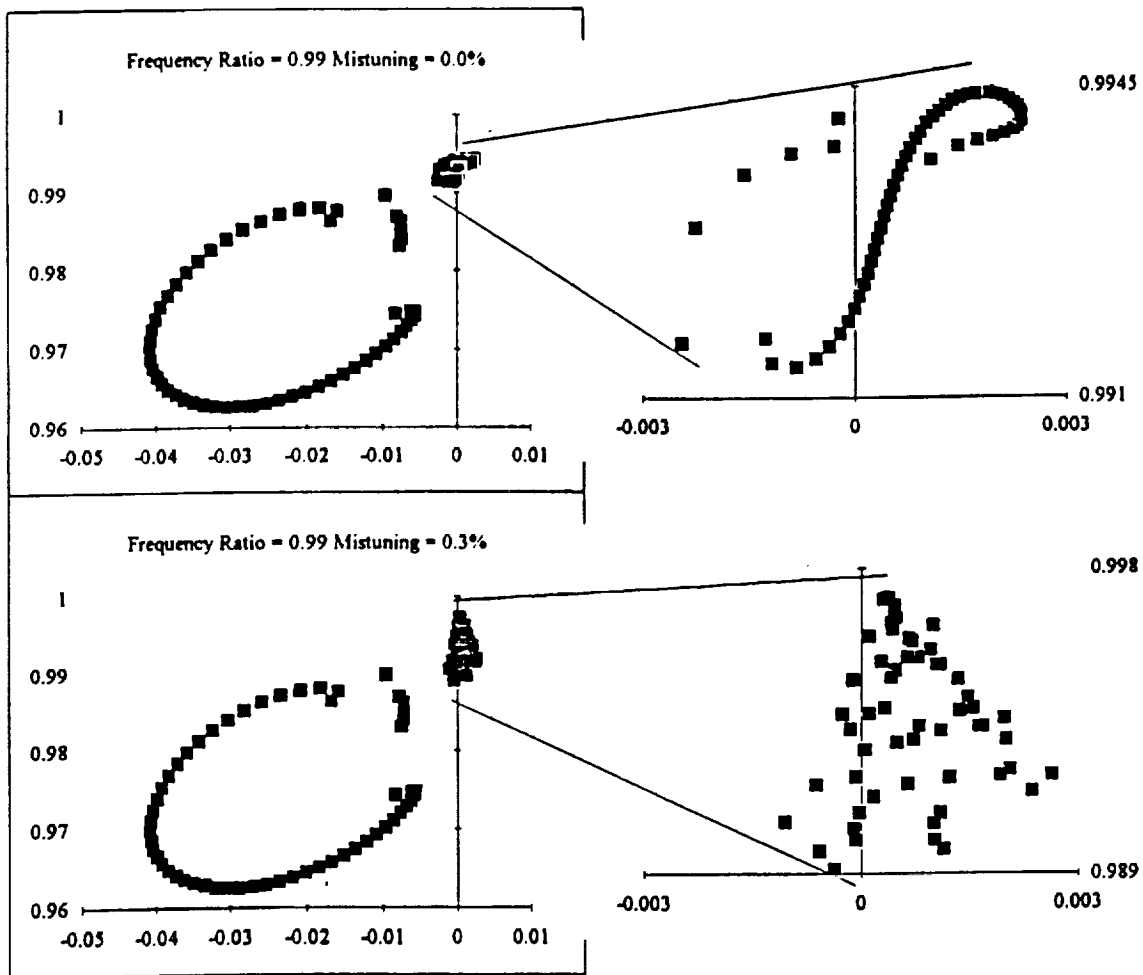
NASA research program thrusts

- Aeroelastic characteristics of mistuned assemblies: mode localization and root locus scattering
- Stochastic measures of sensitivity to mistuning
 - ☐ transfer matrix based
 - ☐ eigenvalue perturbation based
 - ☐ localization factors
 - ☐ composite sensitivity measure for structurally and aerodynamically coupled rotors
- Dynamics of mistuned assemblies with several component modes per blade.
Effect of close blade modes on tuned and mistuned system dynamics.
- Design for low sensitivity to mistuning: formulation of an optimization constraint.
- Forced response of mistuned assemblies:
 - physical mechanisms governing mistuning effects
 - efficient statistical computational methods
- Mistuned bladed disk formulation via component mode analysis and validation of simple models



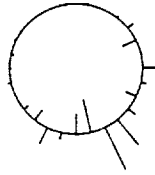






Practical Significance of the Localization Factor

decay according to a - just



$$S_{mid} \approx 25$$

$$\gamma = 0.2$$

90% amplitude decay by the 11th blade

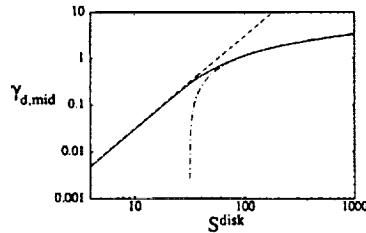
- For $\gamma = 0.1$, amplitude decays by a factor $e^{-0.1} \approx 0.9$ from one bay to the next (on *average*)

56% of the energy is transmitted to the 3rd bay

- For $\gamma = 1.0$, average energy transmitted to next bay is 13.5% and less than 0.25% of the energy reaches the 3rd bay!

- γ is an average quantity and specific realizations of mistuned systems may exhibit different behavior.

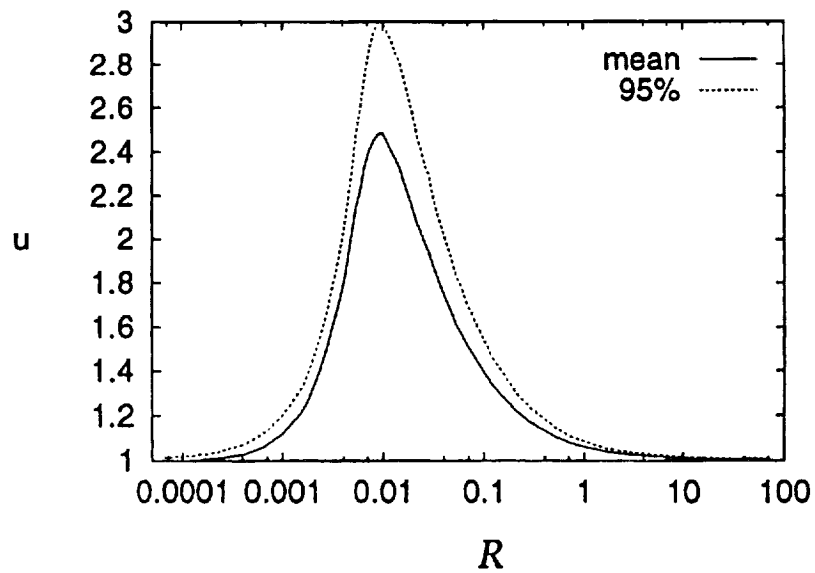
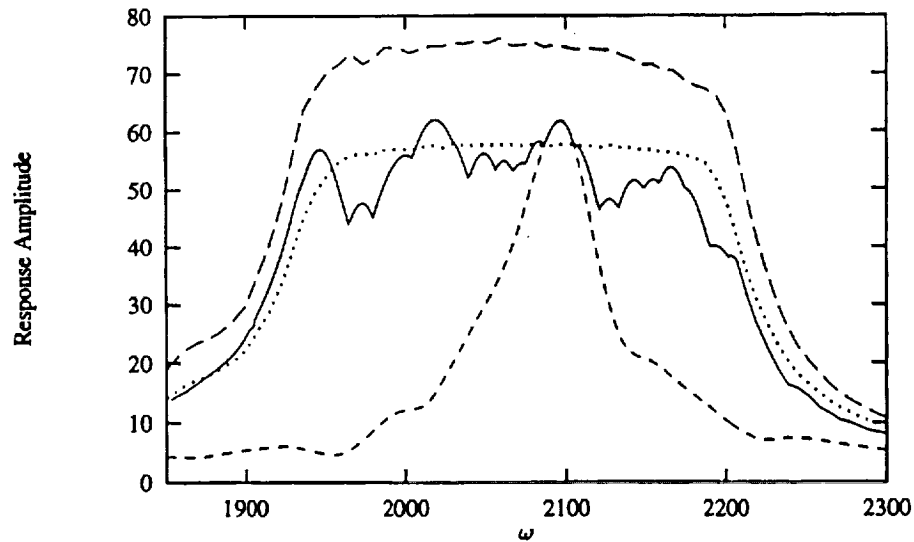
- γ can be calculated in terms of a universal sensitivity measure for simple models.



- Use in design:

Maximum allowable localization strength $\Rightarrow \gamma \Rightarrow S \Rightarrow$ corresponding permitted regions in parameter space.

Forced Response of Mistuned Assemblies



$$u = \frac{\text{Maximum blade amplitude in mistuned system}}{\text{Blade amplitude in tuned system}}$$

Closing

- Because of its potentially catastrophic effects such as single blade failure, mistuning must be accounted for in the design and analysis of blade assemblies
- Simple and effective mistuning capability must be incorporated into FREPS
- Underlying physical mechanisms must be understood to generate proper reduced-order models

Future work:

- Forced response: develop physical understanding and associated efficient computational techniques
- Mistuning experiment: corroborate occurrence of localization and high sensitivity in nonrotating/rotating conditions
- Beneficial mistuning patterns: practical only if mistuning can be controlled